REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
06-05-2015	Final Report	20-Jul-2009 - 19-Jul-2012

4. TITLE AND SUBTITLE	
Final Report: Evolution and Survival of Quantum Entanglement	5a. CONTRACT NUMBER W911NF-09-1-0385
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER 611102
6. AUTHORS J.H. Eberly	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES	8. PERFORMING ORGANIZATION REPORT NUMBER
University of Rochester 518 Hylan Building, RC Box 270140 Rochester, NY 14627 -0140	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211	10. SPONSOR/MONITOR'S ACRONYM(S) ARO 11. SPONSOR/MONITOR'S REPORT NUMBER(S) 56228-PH-OC.16

12. DISTRIBUTION AVAILIBILITY STATEMENT

Approved for Public Release; Distribution Unlimited

13. SUPPLEMENTARY NOTES

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14. ABSTRACT

Theoretical foundations for methods to preserve quantum entanglement are explored and explained. These engage modifications and extensions and improvements of Markovian and perturbative and near-resonant approximations.

15. SUBJECT TERMS

quantum entanglement, decoherence, qubit, revival, survival, Jaynes-Cummings, Rabi, rotating wave approximation, non-classical

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Joseph Eberly
UU	UU	UU	UU		19b. TELEPHONE NUMBER 585-275-4576

Standard Form 298 (Rev 8/98)
Prescribed by ANSI Std. Z39.18

Report Title

Final Report: Evolution and Survival of Quantum Entanglement

ABSTRACT

Theoretical foundations for methods to preserve quantum entanglement are explored and explained. These engage modifications and extensions and improvements of Markovian and perturbative and near-resonant approximations.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
04/29/2015	6.00	X-F Qian, C J Broadbent , J H Eberly. Bell Violation for Unknown Continuous-Variable States, New Journal of Physics, (01 2014): 13033. doi:
04/29/2015	8.00	S. M. Hashemi Rafsanjani, , C. J. Broadbent , J. H. Eberly,. Bounding the Entanglement of N qubits with only four measurements, PHYSICAL REVIEW A , (06 2013): 62331. doi:
04/29/2015	7.00	S Agarwal, , S M Hashemi Rafsanjani , J H Eberly. Dissipation of the Rabi Model Beyond the Rotating Wave Approximation, J. Phys. B: At. Mol. Opt. Phys. , (06 2013): 224017. doi:
04/30/2015	11.00	X. -F. Qian , J. H. Eberly, . Initial Conditions and Entanglement Sudden Death, doi:10.1016/j.physleta.2012.08.007Phys. Lett. A 376, 2931 (2012). , (08 2012): 0. doi:
04/30/2015	10.00	J. H. Eberly, S. Agarwal . Witnessing Non-classicality of a Quantum Oscillator State by Coupling it to a Qubit , Physical Review A (accepted), (02 2012): 22341. doi:
04/30/2015	12.00	Xiao-Feng Qian. Pre-management of disentanglement , J.Phys. A, (07 2014): 265304. doi:
05/04/2015	13.00	S. M. Hashemi Rafsanjani, M. Huber, C. J. Broadbent, J. H. Eberly. Genuinely Multipartite Entanglement of N-qubit X Matrices, Phys Rev A, (06 2012): 62303. doi:
05/04/2015	14.00	A. Valdés-Hernández, G. H. Aguilar, P. H. Souto Ribeiro, S. P. Walborn, Jiménez Farías, L. Davidovich, X-. F. Qian , J. H. Eberly. Experimental Investigation of Dynamical Invariants in Bipartite Entanglement, Phys Rev A, (01 2012): 12314. doi:
05/04/2015	15.00	S. Agarwal, S. M. Hashemi Rafsanjani , J. H. Eberly. Tavis-Cummings Model Beyond Rotating Wave Approximation, Phys Rev A, (04 2012): 43815. doi:

TOTAL: **9**

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received	Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Buffalo Workshop on Quantum Computing, Sept.17-18, 2011
invited paper: "Strong Coupling: Qubits without the RWA"
SUNY-Buffalo
Amherst NY

Stevens Symposium on Quantum Open Systems and Quantum Information, May 5-6, 2012
invited paper: "Dimension quantization and Bell violation by classical light beams"
Stevens Inst. of Technology
Hoboken NJ

Third Int'l. Workshop on Q. Ent. Dynamics, Decoherence and Quantum Control, June 12-14, 2012
invited paper: "Small-Dimension Quantization in Classical Entanglement of Light"
Shanghai University
Shanghai, China

Workshop on Frontiers of Quantum Information - June 15-16, 2012
invited paper: "When Malus Entangles with Euclid, Who Wins?"
Fudan University
Shanghai, CHINA

Intl. Symposium Photonics Trends - 2012, Sept. 16-18, 2012
"Classical Entanglement and Optical Polarization "
Changchun Inst. of Optics and Mechanics
Changchun, Jilin, China

518. Heraeus Seminar on Quantum Optical Analogies: a Bridge between Classical and Quantum Physics, Oct. 29-Nov. 1, 2012
invited paper: "Non-deterministic Physics and Bell Violation"
Bad Honnef, GERMANY

Intl. Workshop: Quantum Optics and New Materials (V), 27-30 May, 2013
invited paper: "Indeterministic Classical Fields and Bell Violation"
Beijing Computational Science Research Center
Beijing, China

Informal QOLS Seminar, 31 October, 2013
Non-Quantum Entanglement: what is it and what can it do?
Physics Department / Imperial College
London UK

Number of Presentations: 8.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received		Paper
08/16/2011	2.00	S. Agarwal, , S.M. Hashemi Rafsanjani, , J.H. Eberly. Two Qubits Tavis-Cummings Model Beyond the Rotating Wave Approximation: Degenerate Regime, arXiv:1106.0052 (06 2011)
08/16/2011	3.00	. Exact Entanglement Dynamics Beyond the Rotating Wave Approximation, arXiv:1105.2835 (05 2011)
08/16/2011	1.00	O. Jiménez Farías, , A. Valdés Hernández, , G. H. Aguilar, , P. H. Souto Ribeiro, , S. P. Walborn, , L. Davidovich, , Xiao-Feng Qian, , J. H. Eberly. Experimental investigation of dynamical invariants in bipartite entanglement, arXiv:1103.1840 (08 2011)

TOTAL: **3**

Number of Manuscripts:

Books

Received Book

TOTAL:

Received Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

During the grant period the PI was awarded two honors, with financial prizes, by the Optical Society of America:

1. Frederic Ives Medal with the Jarus Quinn Prize

2. Distinguished Service Award

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
S. Agarwal	0.13	
X.F. Qian	0.16	
FTE Equivalent:	**0.29**	
Total Number:	**2**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
C.J. Broadbent	0.38
Muhammed Yonac	0.29
FTE Equivalent:	**0.67**
Total Number:	**2**

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
J.H. Eberly	0.10	No
FTE Equivalent:	**0.10**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED	Discipline
Joshua Geller	0.05	Physics
FTE Equivalent:	**0.05**	
Total Number:	**1**	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 1.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 1.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 1.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 1.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 1.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME	
S. Agarwal	
X.F. Qian	
Total Number:	**2**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Scientific Progress and Accomplishments

Control of entanglement was the principle goal. Success means the discovery and quantification of ways to predict and manipulate entanglement. Several notable successes in our pursuit of this goal can be identified. Two of the participating PhD students were able to describe maximizing genuine multipartite entanglement of N mixed qubits. A complementary result with immediate connection to ongoing experimental work in the Rainer Blatt group is a prescription for bounding the entanglement of N qubits with only four measurements. This is a form of entanglement witness, and another witness operation was discovered to determine non-classicality of an oscillator state by coupling to a qubit. Degree of quantum character plays an important role in using entanglement and we extended the use of Bell Inequality violation as a measure of this by prescribing a Bell violation procedure applicable to continuous variable states. Another connection to experimental work (done by the Rio de Janeiro team of Davidovich and Souto Ribeiro) was our prediction of dynamical invariants of bipartite entanglement.

Finally, we were able to identify and describe quantitatively a situation (not yet within experimental reach) where four important aims can be realized together: external control of entanglement that is coherent, that applies to an arbitrary number N of qubits, that applies to mixed as well as pure states, and that allows the degree of surviving entanglement to be determined.

Technology Transfer

Evolution and Survival of Quantum Entanglement

J .H. Eberly, University of Rochester

ARO-W911NF-0910385 Final Report

Our work is both theoretical and mathematical, and is designed to encourage and assist experimental work in advancing toward the overall goal of quantum entanglement preservation, throughout performance of protocols developed independently for tasks of quantum information. These include quantum computing, quantum cryptography, quantum teleportation and other forms of entanglement swapping. Fundamental experimental activities concerning entanglement operations under ambient noise, including entanglement swapping, quantum feedback, control of many-body coherence, and time-dependent disentanglement on different time scales, can be related to our work.

One obvious goal is to promote control of the effect of noise processes on quantum operations. We have been successful in demonstrating that a small set of controllable parameters, in specifying initial quantum states, can provide valuable information about a certain decoherence phase space. This allows separation of outcome trajectories into three categories. These have immediate laboratory relevance. One category contains the compact collection of initially entangled states that can be predicted to lead to rapid destruction of entanglement under common noisy environments. A second category comprises the states that are isolated from sudden entanglement death and permit indefinitely long survival times during slow decay, and the third, probably most interesting category, comprises the states that will suffer entanglement loss in a finite time, but for which the end of entanglement can be guaranteed to occur no sooner than a fixed time. This by itself is a new finding.

A specific challenge in treating entanglement evolution and survival is to extend well-known results beyond the validity of the rotating wave approximation (RWA). This is largely unknown territory but is becoming practically necessary as qubit-qubit entanglement is probed and controlled by stronger and stronger interactions. The Tavis-Cummings model is ideal for a careful exploration of this new domain, as it incorporates more than one qubit interacting with a common oscillator mode. We have treated the parameter regime in which the frequencies of the qubits are much smaller than the frequency of the associated cavity oscillator. We have extended the adiabatic approximation introduced by Irish for a single-qubit system to the multiqubit case.
For example, in a two-qubit system, we identified three-state manifolds of close-lying dressed energy levels and obtained results for the dynamics of intramanifold transitions that are not known in the traditional regime of the RWA. Both number state and coherent state preparations were examined, and we derived analytical formulas that simplify the interpretation of numerical calculations. Expressions for individual collapse and revival signals of both population and entanglement were obtained.

Nonclassicality of system character is of course essential, and we found a new witness operation that guarantees nonclassical character of a quantum field state. Importantly, the method does not require state reconstruction and a bound is established for the evolution of a qubit which is coupled to the oscillator. Violations of the bound are an igorous signal of the nonclassical character of the initial oscillator state. Nonclassicality is also monitored via Bell violation, but this method has distinct limitations, being proved only for systems with discrete quantum states (typically only two states per system). We have found how to extend this criterion for non-classicality to a pair of systems, only one of which is restricted to be a qubit. However, the use of the Schmidt Theorem of analytic function theory allows one qubit the 'control' the dimensionality of a coupled continuously infinite-dimensional partner and discards all but two dimensions in the continuous manifold. This has the prospect of opening a new domain for Bell violation tests to be carried out.

To return to experimentally directly relevant work, we obtained experimental support for our finding on non-conservation of entanglement. When two or more particles interact, entanglement is set apart from other dynamical quantities, such as energy and momentum, because of its non-conservation, and so does not allow the interpretation of the subtle dynamics of entanglement as a flow of this quantity between the constituents of the system. Nevertheless, in bipartite qubit interactions we showed that the inclusion of a third party leads to a new time-invariant expression. This engages the bipartite entanglement between each of the parties and the other two, reminiscent but not the same as monogamy. Experimental demonstration of this idea using entangled photons has now been made.

Finally it's important to note that entanglement occurs in various 'strengths' and it can be critical to establish not only the presence of entanglement, but to have a bound on its strength. This can be measured systematically for qubits via concurrence, and we have found a concurrence-based measure for the genuinely N-partite (all-party) entanglement of N-qubit states. This uses the trace distance metric and leads to a new algebraic formula for the Greenberger-Horne-Zeilinger (GHZ)-diagonal states. We then showed how the all-party entanglement of GHZ states produced by the Rainer Blatt quantum computing team (Innsbruck) for an arbitrary number of qubits may be bounded with only four measurements. This we expect to see applied widely.